Carolina Dynamics Symposium - Titles and Abstracts

Furman University

Friday, April 22nd - Sunday April 24th, 2022

Claire Merriman – Friday 4:00 pm

What do fractions, dynamical systems, and geometry have to do with each other?

Abstract:

Continued fractions are a way to represent a real number as $a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \cfrac{1}{\ddots}}}$. They appear in various mathematics, from number theory to dynamical systems to geometry. I will explain how these continued fractions give nice description of paths on geometric surfaces, somewhat like describing the slope of a line. One way to connect the number theoretic continued fractions to the geometric surfaces is by use a dynamical system, basically, a function that is applied repeatedly to determine the continued fraction expansion. The geometric properties of these pictures help us to describe patterns in the continued fraction expansions, and the continued fraction expansions provide a compact description of the geometry.

Scott Schmieding – Saturday 9:30 am

Genericity for subshifts

Abstract:

I'll discuss some recent work on the structure of generic subshifts within various spaces of subshifts with the Hausdorff topology. This is joint work with Ronnie Pavlov.

Robert Bland – Saturday 10:30 am

Subsystem entropies of SFTs and sofic shifts on countable amenable groups

Abstract:

In symbolic dynamics, a shift of finite type (SFT) is a dynamical system which is completely specified by a finite, combinatorial rule, and a sofic shift is a topological factor of an SFT. In general, one would like to understand the structure of the subsystems of a given SFT or sofic shift, especially through the lens of entropy theory. In this direction, much is classically known for systems over $\mathbb{Z}$. Less is known for systems over $\mathbb{Z}^d$ for $d > 1$, and even less is known with full generality for systems over arbitrary countable amenable groups. In this talk, we present a theorem: if X is an SFT over G, an arbitrary countable amenable group, and $h(X)$ is the (topological) entropy of X, then the entropies of the SFT subsystems of X are dense in the interval $[0, h(X)]$. An analogous result is obtained for sofic shifts. The structure of our proof comes from earlier work of Desai in the case of systems over $\mathbb{Z}^d$, and the generalization is made possible by recent work of Downarowicz, Huczek, and Zhang on tilings of countable amenable groups. Joint work with Kevin McGoff and Ronnie Pavlov.

Wenbo Sun – Saturday 11:15 am

Recent advances in jointly ergodic problems

Abstract:

The mean ergodic theorem states that the average of a function under the iterations of an ergodic transformation converges to the integral of the function. It is a natural question to ask under what conditions, does the average of the product of multiple functions under different iterations of transformations converge to the product of the integrals of the functions. Questions of this type are called jointly ergodic problems. In this talk, I will introduce the developments on this topic, as well as some recent advances based on the joint works with S. Donoso, A. Ferre and A. Koutsogiannis.

Yun Yang – Saturday 1:30 pm

Entropy rigidity for 3D Anosov flows

Abstract:

Anosov systems are among the most well-understood dynamical systems. Special among them are the *algebraic systems*. In the diffeomorphism case, these are automorphisms of tori and nilmanifolds. In the flow case, the algebraic models are suspensions of such diffeomorphisms and geodesic flows on negatively curved rank one symmetric spaces. In this talk, we will show that given an integer $k \geq 5$, and a C^k Anosov flow Φ on some compact connected 3-manifold preserving a smooth volume, the measure of maximal entropy is the volume measure if and only if Φ is $C^{k-\varepsilon}$-conjugate to an algebraic flow, for $\varepsilon > 0$ arbitrarily small. This is a joint work with Jacopo De Simoi, Martin Leguil and Kurt Vinhage.

Ronnie Pavlov – Saturday 2:30

Word complexity and automorphism groups for subshifts

Abstract:

Two ways of measuring the "size" of a (one-dimensional) subshift X are its word complexity function $c_X(n)$, which counts the number of words of any length n appearing in sequences in X, and its group $\mathrm{Aut}(X)$ of automorphisms, which are self-homeomorphisms of X which commute with the shift action.

There are many interesting results in recent years supporting the heuristic that subshifts with "low complexity" (meaning $c_X(n)$ grows slowly) should have "small" automorphism group. I will summarize some of these results and describe a new one (joint with Scott Schmieding): if $c_X(n)$ grows more slowly than $n(\log\log n)^C$ for every $C > 0$, then $\mathrm{Aut}(X)$, after modding out by the subgroup generated by the shift, is locally finite.

Akshat Das – Saturday 3:15 pm

An adelic version of the three gap theorem

Abstract:

In order to understand problems in dynamics which are sensitive to arithmetic properties of return times to regions, it is desirable to generalize classical results about rotations on $\mathbb{R}/\mathbb{Z}$ to rotations on adelic tori. One such result is the three gap theorem (or Steinhaus conjecture). It states that, there are at most three distinct gap lengths in the fractional parts of the sequence $\alpha, 2\alpha, \ldots, N\alpha$ for any $\alpha \in \mathbb{R}$ and $N \in \mathbb{N}$. One of the more recent proofs of this has been given by Marklof and Strömbergsson, in which they use a lattice based approach to gaps problems in Diophantine approximation. In this talk, we adapt this approach to prove a natural generalization of the classical three gap theorem for rotations on adelic tori. This is joint work with Alan Haynes.

Jim Campbell – Saturday 4:30 pm

The Lightning Model

Abstract:

When Carolina Dynamics met at Agnes Scott a few years ago, I presented a crude model for lightning that had been conceived by some freshman honors students. In the meantime, through discussions with Anthony Quas and his student Alexandra Deane, we modified and developed the model more fully. The correct setting is as a non-standard model for a type of directed percolation on the integer lattice $\mathbb{Z}^2$. Randomly assign to each vertex $a \in \mathbb{Z}^2$ a potential, denoted ϕ_a, chosen independently and uniformly from the interval $[0, 1]$. For fixed $\epsilon \in [0, 1]$, draw a directed edge from vertex a to a nearest-neighbor vertex b if $\phi_b < \phi_a + \epsilon$, yielding a directed subgraph of the infinite directed graph $\overrightarrow{G}$ whose vertex set is $\mathbb{Z}^2$, with nearest-neighbor edge set. We define notions of weak and strong percolation for our model, and observe that when $\epsilon = 0$ the model fails to percolate weakly, while for $\epsilon = 1$ it percolates strongly. We show that there is a positive ϵ_0 so that for $0 \leq \epsilon \leq \epsilon_0$, the model fails to percolate weakly, and that when $\epsilon > p_{\text{site}}$, the critical probability for standard site percolation in $\mathbb{Z}^2$, the model percolates strongly. We show that for these 'directed percolation' models, there are some subtle issues that do not arise for undirected percolation. Although our model does not have the finite energy property, we are able to show that, as in the standard model, the number of infinite strongly connected clusters is almost surely 0, 1 or ∞.

Kitty Yang – Sunday 9:00 am

Mapping class group of low complexity subshifts

Abstract:

Let (X, σ) be a subshift. A flow equivalence of two dynamical systems is an orientation-preserving homeomorphism of the suspensions of the systems. The mapping class group of a subshift is the group of self-flow equivalences up to isotopy. Under mild conditions, the automorphism group (modulo$\langle\sigma\rangle$) of a subshift embeds into the mapping class group. We compute the mapping class group for minimal subshifts of linear complexity, specifically interval exchange transformations.

James Kelly – Sunday 10:00 am

Chaos and entropy for linear operators

Abstract:

We examine chaotic behavior for linear operators on Banach spaces. We particularly focus on backward shift operators on the weighted sequence spaces ℓ_v^p and $c_{0,v}$. We give some characterizations for these operators to be Li-Yorke chaotic, and we characterize when they have positive topological entropy. For linear operators more generally, we demonstrate relationships between positive topological entropy and other properties related to chaotic behavior.

Martin Schmoll – Sunday 10:45 am

Siegel-Veech constants for branched cyclic covers

Abstract:

We present the general method to calculate α-area Siegel-Veech constants for branched cyclic covers on translation surfaces by evaluating the Siegel-Veech formula on strata of the moduli space. The result is a formula that depends surprisingly little on the branching data. We will give some background and motivation for this research based on billiards in a polygon.