Stability and Centers in the Moon-Rand Systems

Abstract

In the mid-80's F. C. Moon and R. H. Rand examined a method for control of flexible space structures using feedback control of certain elements of the stiffness matrix. The method leads to nonlinear control equations, and ultimately systems of nonlinear ordinary differential equations in $\mathbb{R}^3$ with an equilibrium at the origin for which the linear part of the system has a pair of purely imaginary eigenvalues for all values of the parameters. Moon and Rand examined one family of such systems and found a sufficient condition for asymptotic stability of the equilibrium and by numerical means evidence for existence of asymptotically stable periodic oscillation. We use this family as a platform for a discussion of methods for examining stability and center conditions for systems of ODE's in $\mathbb{R}^2$ and $\mathbb{R}^n$ whose linear part at an equilibrium has a pair of purely imaginary eigenvalues, using center manifold theory for $n > 2$. We treat both theoretical and computational aspects of the problem, and return to the Moon-Rand families to illustrate the methods.