TIRE TRACKS GEOMETRY, HATCHET PLANIMETER, MENZIN'S CONJECTURE, AND COMPLETE INTEGRABILITY

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ABSTRACT. This talk concerns a simple model of bicycle motion: a bicycle is a segment of fixed length that can move in the plane so that the velocity of the rear end is always aligned with the segment. The trajectory of the front wheel and the initial position of the bicycle uniquely determine its motion and its terminal position; the monodromy map sending the initial position to the terminal one arises. This circle mapping is a Moebius transformation, a remarkable fact that has various geometrical and dynamical consequences. Moebius transformations belong to one of the three types: elliptic, parabolic and hyperbolic. I shall outline a proof of a 100 years old conjecture: if the front wheel track is an oval with area at least Pi then the respective monodromy is hyperbolic. I shall also discuss the related Darboux transformation, in the continuous and discrete settings, and its complete integrability.