STRONG SHIFT EQUIVALENCE OF MATRICES OVER A RING

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Abstract. Let R be a ring. Two square matrices A, B are elementary strong shift equivalent (ESSE-R) over R if there are matrices U, V over R such that $A = RS$ and $B = SR$. Strong shift equivalence over (SSE-R) is the equivalence relation generated by ESSE-R. Shift equivalence over R (SE-R) is a tractable equivalence relation which is refined by SSE-R. The refinement is trivial if $R = Z$ (Williams), a principal ideal domain (Effros 1981) or a Dedekind domain (Boyle-Handelman 1993). No results have appeared since 1993.

It turns out that this refinement is captured precisely by a certain quotient group of the group $NK_1(R)$ of algebraic K-theory. It follows that for very many (not all) rings R, the relations SE-R and SSE-R are the same. For the class of nilpotent matrices over R (nilpotent matrices are those shift equivalent to [0]), this quotient group is $NK_1(R)$ itself. When $NK_1(R)$ is not trivial, it is not finally generated. (Farrell, 1977).

This is joint work with Scott Schmieding.