Bifurcation of Critical Periods

Douglas S. Shafer

Abstract

A number of bifurcational problems in the qualitative theory of systems of polynomial differential equations on $\mathbb{R}^2$,

$$\dot{x} = P(x, y), \qquad \dot{y} = Q(x, y), \tag{1}$$

can be formulated as that of obtaining an upper bound on the number of positive isolated zeros that can appear in a neighborhood of the origin when a real analytic family of real analytic functions of the form

$$\mathcal{F} : \mathbb{R} \times E \subset \mathbb{R} \times \mathbb{R}^m \to \mathbb{R} : (z, \theta) \mapsto \sum_{j=0}^{\infty} f_j(\theta) z^j$$

is perturbed from a parameter string θ^* at which $f_j(\theta^*) = 0$ for all j. The most famous example is the bifurcation of limit cycles from a center of a polynomial system (1) in which z is the radial polar coordinate r and θ is any admissible string of coefficients in the right hand sides P and Q.

A method of Bautin for treating such problems, which involves obtaining a "minimal" finite basis for a particular ideal in the polynomial ring with indeterminates the admissible coefficients in P and Q, can be extended to the problem of critical periods: for a system (1) with a center at the origin for which the period function is $T(r)$, we consider the maximum number of of zeros of the derivative T' that can bifurcate from the zero of T' at $r = 0$ under small perturbation. The method is now feasible because of advances in both computer hardware and methods of computational commutative algebra.

[Joint work with Brigita Ferčec (Maribor), Viktor Levandovskyy (Aachen), and Valery Romanovski (Maribor).]