

# 1/2-heavy sequences driven by rotation

David Ralson

## Abstract

We will examine what could reasonably be called the simplest transformation (irrational rotation of the circle  $\mathbb{R}/\mathbb{Z}$ ) coupled with perhaps the simplest nonconstant function ( $f(x) = 1$  if  $x \leq 1/2$ ,  $f(x) = -1$  if  $x > 1/2$ ). Our primary goal will be to understand the structure and Hausdorff dimension of the (nonempty!) set of  $x$  for which the ergodic sum  $S_n(x) = f(x) + f(x + \theta) + \cdots + f(x + (n - 1)\theta)$  is nonnegative for *every*  $n \in \mathbb{N}$ . Secondary topics may include relating this question to the study of discrepancy sums, a.k.a. diffusion rates of the geodesic flow on a particular infinite surface.