Carolina Dynamics Symposium 2017

- **Nic Ormes.**
  Directed Graphs, Symbolic Dynamics and the Nonnegative Inverse Eigenvalue Problem

  The Nonnegative Inverse Eigenvalue Problem (NIEP) asks for necessary and sufficient conditions for a list of complex numbers to be the eigenvalues of a matrix with nonnegative entries. This problem was posed in 1949 by H. R. Suleimanova and remains open today. In this talk we will discuss variants of this problem which have been solved in papers of Boyle-Handelman (1991) and Kim-Ormes-Roush (2000). The talk will focus on the connections between nonnegative matrices, directed graphs and symbolic dynamics which were critical to these results.

- **Sarah Koch.**
  Postcritical sets in complex dynamics

  In complex dynamics, we are typically interested in iterating a rational map f on the Riemann sphere and studying the orbits of different points under iteration. The orbits of the critical points of f play a very important role in this process. Let $P(f)$ denote the *postcritical set* of f; that is, the union of the orbits of the critical points. Of particular interest is when $P(f)$ is a finite set, in which case f is said to be *postcritically finite*. In this talk, we study the subsets of the Riemann sphere that arise as $P(f)$ for some postcritically finite rational map f. We employ a variety of results to explore this problem, ranging from Belyi's celebrated theorem, to analytic techniques used in the proof of Thurston's topological characterization of rational maps, a central result in complex dynamics. This talk is based on joint work with L. DeMarco and C. McMullen.

- **Isaac Garcia.**
  The 3-dimensional analytic zero-Hopf singularity: center problem, integrability and normal forms

  We show that the well-known Poincaré-Lyapunov nondegenerate analytic center problem in the plane and its higher dimensional version expressed as the 3-dimensional center problem at the zero-Hopf singularity have a lot of common properties. In both cases the existence of a neighborhood of the singularity in the phase space completely foliated by periodic orbits (including equilibria) is characterized by the fact that the system is analytically completely integrable. Hence its Poincaré-Dulac normal form is analytically orbitally linearizable. Also there exists an analytic Poincaré return map and, when the system is polynomial and parametrized by its coefficients, the set of systems with centers corresponds to an affine variety in the parameter space of coefficients. Some quadratic polynomial families are considered.

- **Christian Wolf.**
  Ground States at the Bounary of Rotation Sets

Ground states are accumulation points of equilibrium states when the temperature goes to zero. They play a fundamental role in statistical physics. In this talk we consider rotation sets associated with a continuous dynamical system on a compact metric space and a m-dimensional continuous potential. We study the question for which boundary values one can realize an entropy maximizing measure in its rotation class as a ground state associated with a certain linear combination of the potential. We show that at an exposed point there always exists a ground state that maximizes entropy in its rotation class. We also construct examples of rotation sets (in any dimension m) that have exposed boundary points without a converging ground state in its rotation class. Finally, we consider non-exposed points and show that the following two phenomena exist: a) boundary points without an associated ground state; b) boundary points with a converging ground state that is not ergodic.

- **Natalie Priebe Frank.**
  A family of one-dimensional substitution tilings with infinite local complexity

  In this talk we introduce a parameterized family of substitution tilings on the line that generalizes the Fibonacci substitution. For most values of the parameter the tiling dynamical system is minimal, uniquely ergodic, and has countable Lebesgue spectrum.

- **Robbie Robinson.**
  Technicolor Sturmian sequences and Ostrowski expansions.

  A well known result is that every positive integer is uniquely expressible as a sum of Fibonacci numbers where no two adjacent Fibonacci numbers are ever used. Ostrowski expansions of positive integers generalizes this result by replacing the Fibonacci "scale" with the denominators of the continued fraction convergents of an arbitrary irrational alpha in $[0, 1)$. It is easy to find this expansion using a greedy algorithm (finding the most significant digit first), but for ordinary radix expansions, there is a second algorithm that finds the digits starting from least significant. In this talk we describe a similar algorithm for the Ostrowski expansion that obtains the digits in increasing order of significance in terms of the codings irrational rotations. These codings are not quite the usual "black and white" Sturmian sequence, but a related sequence on a larger "technicolor" alphabet. The proof uses the real Ostrowski expansion, which is a generalization of beta expansion.This is Joint work with Avraham Bourla of Brandeis.

- **Sarah Frick.**
  Essentially faithful codings for a family of limited scope adic transformations

  Limited scope Bratteli diagrams have an increasing number of vertices at each level, however, the number of edges leaving a particular vertex is restricted. We show that for every possible edge ordering, the adic exhibits a coding for which there is a set of measure 1 on which the coding is faithful.

- **Dennis Pace.**
  Surface entropy in $\mathbb{Z}^2$ subshifts

We define the notion of surface entropy, an extended real number associated with a $\mathbb{Z}^2$ subshift that, when paired with entropy, provides a more accurate prediction of the word count. We then discuss the behavior of surface entropy under factor maps, namely that surface entropy is not a conjugacy invariant and how it can be extended to one. Surprisingly, the eccentricity of the rectangles used to count words affects surface entropy; we discuss this relationship and how surface entropy varies as a function of eccentricity. Finally we discuss the class of numbers that can be realized as the surface entropy of a $\mathbb{Z}^2$ SFT.

- **Ronnie Pavlov.**
  Non-uniform mixing and intrinsic ergodicity for subshifts.

  In topological dynamics, a wide range of mixing properties are studied. Standard topological mixing is the ability, given two open sets, to find a point which lies in the first and then, after some number of iterates, lies in the second. In the setting of symbolic dynamics, this can be interpreted as the ability, given two (say n-letter) words in the language of a subshift, to place some gap in between which can be filled to make a new word in the language.

  When the gap can be chosen with length independent of the words to be combined, we have the celebrated specification property of Bowen which, among other properties, implies intrinsic ergodicity, i.e. uniqueness of the measure of maximal entropy.

  When the required gap has length $f(n)$ dependent on the length n of the words to be combined, it is natural to wonder whether slow growth of $f(n)$ might also yield positive properties. I will summarize some recent results in this area, mainly focusing on intrinsic ergodicity.

- **Ayşe Şahin.**

  TBA

- **Jim Wiseman.**
  Recurrence for powers and products.

  For a continuous map f of a compact metric space, we consider different topological recurrent sets $R(f)$ (for example, the nonwandering set, the chain recurrent set, or the generalized recurrent set). We examine the relationships among $R(f)$, $R(f^k)$, and $R(f \times g)$, as well as the connections among the different types of topological recurrence.

- **Richard Neidiner.**
  An arbitrary-order Taylor series method DE solver in MATLAB

  We introduce methods behind and applications of a new numerical solver for initial value problems given by any ordinary differential equations (DE) of the (vector) form $Y' = f(t, Y)$. It is really the MATLAB implementation that is new, since arbitrary order Taylor series methods are known to be theoretically possible but often thought to be impractical. Runge-Kutta approximations of series solutions of orders four or five are an industry standard. However, this MATLAB solver uses arbitrary order, 20 or 30 for example, which enables huge step sizes. By overloading operators to combine series coefficients (not just approximations), the DE itself becomes a recurrence relation. We show how this effective modern (automatic differentiation) approach generalizes the simple recurrence relations sometimes done laboriously by hand in an introductory DE course. The method uses simple polynomial systems to handle any transcendental functions. The solution of a chaotic system for a forced damped pendulum is shown to compute reliable orbits in a very small fraction of the number of steps of built-in MATLAB solvers. The method shows promise for automatically bounding the theoretical error, not just comparative error, on each numerical step.

- **Drew D. Ash.**
  Topological Speedups, Jump Functions, and Orbit Equivalence

  Given a dynamical system (X, T) one can define a speedup of (X, T) as another dynamical system $S : X \to X$ where $S(x) = T^{p(x)}(x)$ for some $p : X \to \mathbb{Z}^+$. In 2015, the speaker gave necessary and sufficient conditions for a minimal Cantor system to be a topological speedup of another minimal Cantor system in terms of known orbit equivalence invariants. Specifically, characterizing topological speedups in terms of associated dimension groups and simplices of invariant probability measures. In this talk, which is a conglomeration of results both individual and with Lori Alvin and Nic Ormes, we will focus on how these invariants can or cannot change depending on particular systems and various continuity conditions on our "jump function" p. For example, using an unbound jump function we can speedup the dyadic odometer to a system with positive entropy, whereas every speedup of the dyadic odometer using a bounded (hence continuous) jump function is conjugate to the dyadic odometer.

- **Karl Petersen.**
  Expansive Adics.

  Which dynamical systems defined by cutting and stacking (equivalently adic systems presented by ordered Bratteli diagrams) can be coded as subshifts? We say such a system is essentially expansive if there is a k such that the partition by the levels of the towers at stage k (equivalently the partition of the space of infinite paths determined by the first k edges) generates the full sigma-algebra for every fully-supported finite ergodic invariant measure. In previous work with Sarah Bailey Frick and Sandi Shields, extending a result of Xavier Méla, we showed that every ordering of the Pascal graph produces an essentially expansive system. In joint work with Terry Adams and Sébastien Ferenczi, we show how to reorganize every odometer to produce essential expansiveness, completing the equivalence of the various concepts of rank one. Our sufficient condition for essential expansiveness applies to all cutting and stacking (or adic) systems and thus may be useful in other situations where a measure-theoretic isomorphism to a subshift would allow the application of ordinary symbolic methods to study dynamical properties.