CAROLINA DYNAMICS SYMPOSIUM 2018 – ABSTRACTS

Sarah Day, College of William and Mary: *Computational Topology and the Life Sciences: Finding structure in models and data*

The field of topology, and in particular computational topology, has produced a powerful set of tools for studying both model systems and data measured directly from physical systems. I will focus on three classes of topological tools: computational homology, topological persistence, and, very briefly, Conley index theory. To illustrate their use, I will discuss recent projects studying coupled-patch population dynamics, flickering red blood cells, and pulse-coupled neurons.

Doug Shafer, UNC Charlotte: *Asymptotics and Stability of the Delayed Duffing Equation*

We examine how insertion of a delay in a system of ordinary differential equations can change the asymptotic dynamics and stabilize the structure of the system under further change, with particular attention to the creation of limit cycles. (Joint work with David C. Hill)

Jane Hawkins, UNC Chapel Hill: *Bifurcations vs. stability in parameter space of iterated meromorphic functions*

We iterate some doubly periodic meromorphic functions with poles exactly at the corners of the squares that define the lattices. The functions we study are of the form $\wp_\Lambda + c$ where $\wp_\Lambda$ is the Weierstrass elliptic $\wp$ function, and c is any complex constant. $\wp$ is the doubly periodic analog of the singly periodic sine function - the building block which, along with its derivative, generates all doubly periodic meromorphic functions.

We parametrize the dynamics by varying both the lattice and the constant, and show regions in which both stability and instability arise. Since there are two complex parameters involved, (one for the square lattice and one for the constant), we only visualize slices of space. While the definitions are obvious extensions of those used for maps like $z^2 + c$, the results are quite different. Some results hold more generally beyond square lattices.

Joseph Rosenblatt, IUPUI and UIUC: *Quantization and shrinking targets for dynamical systems*

Using dynamical systems to create good quantization leads naturally to questions about shrinking targets. A variety of results and some open questions will be presented.

Sarah Frick, Furman: *Periodic Codings of Some Ergodic Systems*

We consider a class of systems given by cutting and stacking which exhibit periodic codings when coded according to levels of the towers at some stage. We give the systems in terms of their cutting and stacking representations as well as their Bratteli diagram representations. We examine conditions under which the codings by every level are periodic.

Claire Merriman, Illinois: *Using modular surfaces to generate continued fractions*

Continued fractions are frequently studied in number theory, but they can also be described geometrically. I will give both pictorial and algebraic descriptions of the cutting sequences and geodesic flows that describe continued fraction expansions.

This talk will focus on converting from regular continued fractions $n_0 + \frac{1}{n_1 + \frac{1}{n_2 + \dots}}$ to continued fractions of the form $2k_0 + 1 \pm \frac{1}{2k_1 + 1 \pm \frac{1}{2k_2 + 1 \pm \dots}}$, and how this changes the modular surface and cutting sequences that generate the respective continued fraction expansions. This is based on joint work with Florin Boca.

Eric Roberts, University of California, Merced: *Chaotic Advection in Active Nematics*

The recent surge of research into active materials is an exciting development in soft matter physics. Unlike traditionally studied fluids, active fluids are not in equilibrium. Instead, they continuously consume energy to generate internal motion, which can subsequently produce large-scale flows and rich emergent dynamical structures. These moving defects can wind around one another to generate chaotic mixing. We report here on experimental and theoretical work on a biologically inspired active nematic liquid crystal. Densely packed microtubules slide antiparallel to each other at a controlled rate due to kinesin molecular motors. The resulting chaotic advection is studied experimentally using the tools of particle tracking, particle image velocimetry, and fluorescence imaging of labeled tracers. Experimental data are analyzed and interpreted in the context of topological dynamics, thereby bridging the fields of chaotic advection and active fluids. We focus on the topological entropy, measured from the braiding of tracer trajectories, and on the local Lyapunov exponent, measured from the divergence of neighboring tracers.

Nic Ormes, University of Denver: *Symbolic systems of linear complexity*

In this talk we will consider symbolic dynamical systems (X, σ) and the complexity function $c_n(X)$ which counts the number of words of length n which appear in X. The Morse-Hedlund Theorem states that if there is an n such that $c_n(X) \leq n$ then X is a finite set of periodic points. On the other hand, there are examples of infinite minimal subshifts with $c_n(X) = n + 1$ (Sturmian subshifts). It is also easy to construct subshifts with $n < c_n(X) < n + const$ by concatentating two one-sided periodic sequences and then considering the orbit closure of this point. The speaker will discuss a new result (joint with R. Pavlov): If X is a transitive system which is not minimal or created by the aforementioned concatenation process then $\limsup c_n(X) - \frac{3}{2}n = \infty$. We will also see that there are examples which demonstrate the sharpness of this result. Time permitting, we will discuss continuing work (joint with R. Pavlov and A. Dykstra) about relationships between $c_n(X)$ and the possible number of minimal components in X.

Lorelei Koss, Dickinson College: *A family of degree d rational maps with a superattracting fixed point at infinity*

We analyze the dynamics of the family of non-polynomial rational maps, $f_{a,d} = a(z+1)^d/z$, for $a \in \mathbb{C}^* = \mathbb{C} \setminus \{0\}, d \geq 3$. For each d, $\{f_{a,d}\}$ is a family of rational maps of degree d of the Riemann sphere parametrized by $a \in \mathbb{C}^*$. For each function in this family, infinity is a superattracting fixed point that attracts all critical points except possibly one simple critical point. We discuss conditions under which the Julia set is Cantor, connected, or Sierpinski. This is a preliminary report on joint work with Joanna Furno.

Karl Petersen, UNC Chapel Hill: *Tree shift complexity*
A tree shift consists of all labelings of the vertices of a tree by elements of a finite alphabet that omit all of a prescribed (possibly infinite) set of finite patterns. These are exactly the closed sets that are invariant under the shifts associated with the tree. Introduced by Aubrun and Bal and studied also by Ban and Chang, they share properties of the familiar one-dimensional subshifts of symbolic dynamics while preserving a directional aspect that may make them easier to analyze than higher-dimensional subshifts, where questions of undecidability and computability arise. In joint work with Ibrahim Salama, we study the complexity function of a tree shift, which counts as a function of n the number of different labelings of a shape of size n. We define the entropy in a different way than Ban and Chang, prove that the limit in the definition exists, and show that the entropy of a tree shift determined by adjacency constraints dominates the entropy of the associated one-dimensional subshift.

Sarah Day, College of William and Mary: *Combinatorial representations of dynamics: structure and computer-assisted proofs*
Computational Conley index theory has given rise to a number of computer-assisted proof techniques for dynamical systems governed by iterated maps. Recent work with Bill Kalies has sought to extend these techniques in order to broaden the class of systems that may be studied in this manner. This talk will focus on the chosen *combinatorial representation*, the basic structure used to encode dynamics for a computational study. Sample results for the Henon map will serve as illustrations.

Albert Fathi, Georgia Tech: *Recurrence on abelian cover. Application to closed geodesics in manifolds of negative curvature*
If h is a homeomorphism on a compact manifold which is chain-recurrent, we will try to understand when the lift of h to an abelian cover (i.e. the covering whose Galois group is the first homology group of the manifold) is also chain-recurrent. This is related to the proof by John Franks of the Poincaré-Birkhoff theorem. It has new consequences on density of classes of closed geodesics in a manifold of negative curvature.

Howie Weiss, Georgia Tech: *Collateral sensitivity of antibiotic-resistant microbes: Modeling insights informing in-vitro studies*
Antibiotics have greatly reduced the morbidity and mortality due to infectious diseases. Antibiotic resistance constitutes a significant threat to human health. One strategy to help combat resistance is to find novel ways to use existing drugs,

even those that display high rates of resistance. For the pathogens *E. coli* and *P. aeruginosa*, pairs of antibiotics have been identified for which evolution of resistance to drug A increases sensitivity to drug B and visa versa. These research groups have proposed cycling such pairs to treat infections, similar treatment strategies are being investigated for various cancer forms as well.

While an exciting treatment prospect, no cycling experiments have yet been performed with consideration of pharmacokinetics (PK) and pharmacodynamics (PD). To test the plausibility of this scheme and search for ways to optimize it, we create a mathematical model with explicit PK-PD considerations. We study several possible treatment protocols using pairs of collaterally sensitive antibiotics, and investigate the speed of ascent of multiply resistant mutants.

Joanna Furno, Houston: *The Natural Extension for a p-adic Beta-Shift*

The beta-shift on the p-adic integers was defined and studied by d'Ambrose, Everest, Miles, and Ward in 2000 and by Scheicher, Sirvent, and Surer in 2015. We will give the definition, briefly review some of their results, and discuss a simple description of the natural extension.

Julie Barnes, Western Carolina: *Using concepts from complex dynamics to create coloring book images*

There are many beautiful images that can be seen through complex dynamics, especially when studying Julia sets. In this talk, we look for interesting, colorable images that are intricately related to Julia sets, but are not Julia sets themselves. Our procedure is rather simple: find a function, iterate it a few times, compute the real part, and plot contour plots. The tricky part, however, is knowing where to find the functions in the first place. We will explore how plots created in this manner are related to Julia sets, how the shapes of contour plots are prescribed by critical points and poles, and what happens to these plots under iteration. We will discuss the math behind the key artistic components that appear and briefly address how to find functions that produce interesting coloring book images.

Andy Parrish, Eastern Illinois: *Pointwise convergence of ergodic averages: subsequences and group actions*

While the Pointwise Ergodic Theorem ensures convergence of ergodic averages for all integrable functions, it requires the observations in the time averages to be taken at very regular intervals. The behavior of the average becomes much less certain when one considers only observations taken at increasingly rare instances. While several examples show that convergence can still occur, this convergence can be compromised by seemingly inconsequential changes to the sequence. We will explore these examples and discuss how they can be extended to the more general settings of virtually nilpotent and amenable group actions.

James Campbell, University of Memphis: *Lightning strikes!*

Lightning Strikes!

We will explore a discrete model for the formulation of lightning. We place randomly generated numbers (*levels*) in each cell of an $m \times n$ grid, creating a *configuration*. Chooosing a starting cell along the top row, we examine the neighboring cells and (i) draw an edge to any neighbor whose level is less than or equal to our current level (such a cell has become *visited*), (ii) list the visited cells in a queue, and (iii) start the process over at the beginning of the queue, proceeding until the queue is empty.

The pictures in Figure 1 were computer generated from this model, with a 50×50 grid, the cell values chosen uniformly from the set $\{0, 1, 2\}$, and the center cell in the top row as the starting point. Each picture corresponds to a different initial distribution of the integers in the cells.

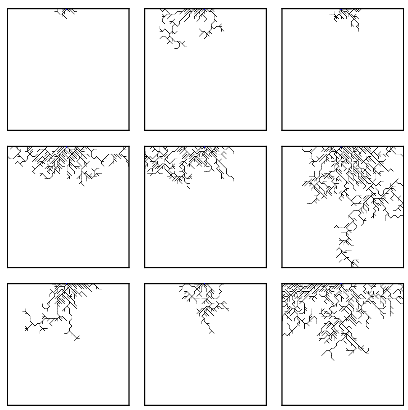

Figure 1: Several simulations on a 50×50 grid; one lightning strike.

We are interested the fate of the resulting path, and would especially like to be able to compute the probability that some portion of the path reaches the bottom of the grid. We think of this case as *success*, or more colloquially, a *lightning strike*.

What sort of student would enjoy this talk?

A student who

- enjoys figuring out different ways of getting somewhere, or
- has a little familiarity with probability, or
- can program a little (even in something like Mathematica or Matlab), or
- enjoys watching lightning in the night sky.